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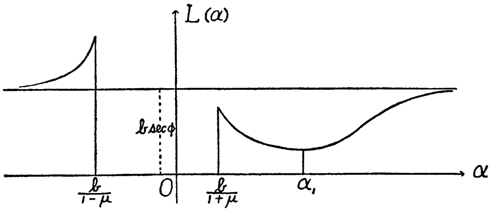

FIG. 3

3. It is readily verified that

$$L\left(\frac{b}{1-\mu}\right) > L(\infty)$$

and

$$L\left(\frac{b}{1-\mu}\right) > L\left(\frac{b}{1+\mu}\right);$$

but that $L[b/(1+\mu)]$ may be greater or less than $L(\infty)$. There is exactly one value of α, viz., α_1, for which $L(\alpha)$ has a minimum; this value α_1 lies between b and $b/2$, so that the shortest circular path from the origin to the line $x = b$ which departs from the origin with slope m lies between the circle which cuts this line perpendicularly and the one which passes through the point $(b, 0)$.

We have seen that $\alpha_1 = b/(1 + \mu \sin \theta_1)$, while θ_1 is the unique root of the equation $\sin\theta + \sin\phi - (\theta + \phi)\cos\theta = 0$ on the interval $(0, \pi/2)$. Hence, θ_1 is determined as a function of ϕ by the relation

$$\frac{\sin\theta_1 + \sin\phi}{\theta_1 + \phi} = \cos\theta_1.$$

From this it follows that $\lim\limits_{\phi \to 0} \theta_1 = 0$; that is, among the circles tangent to the X-axis at the origin, the straight line furnishes the shortest path from the origin to the line $x = b$, which shows that the result obtained above is in accord with known facts.

II. MATHEMATICAL ARTICLES IN ENCYCLOPEDIAS.

BY NORMAN ANNING, University of Michigan.

The efforts of the Association to produce a Mathematical Dictionary that shall be a credit to American scholarship are, in the highest degree, praiseworthy. While that is in progress could not some arrangement be made with publishers who are bringing out important works of reference to have the proofs of mathematical articles read by mathematicians? The editors of the *Encyclopedia Americana* following the policy that articles on technical subjects should be written by experts have, in the main, been singularly happy in their choice of authors. With certain less technical articles they have been less fortunate. For every one who will read through Dr. Webster's excellent treatment of Hydrodynamics there are perhaps a hundred who will consult the *Encyclopedia* to find the "indispensable minimum" of information about the Hyperbola. This is, in part, what the latter will find:[1]

> The length of the transverse axis is said to be a, $\cdots$. The two lines
>
> $$\frac{x}{a} = \pm\frac{y}{b}$$
>
> approach the hyperbola closer than any assigned limit, and are called the assymptotes (sic) of the

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[1] Vol. 14, p. 601.

hyperbola. The points

$$\left(\pm a\left(\frac{\sqrt{a^2+b^2}}{a}-1\right), 0\right)$$

are called foci, $\cdots$. The ratio of the distance from a point on the curve from

$$\left(a\left(\frac{\sqrt{a^2+b^2}}{a}-1\right), 0\right)$$

to its distance from the line

$$x^a = \frac{a^2}{a^2+b^2}$$

has the constant value

$$\frac{\sqrt{a^2+b^2}}{a} \cdots .$$

The lines

$$x = \pm \frac{a^2}{a^2+b^2}$$

are known as directrices.

All in twenty-four lines.

To avoid the charge of partisanship let me add that a glance at the *eleventh* edition of the *Encyclopædia Britannica* reveals, in one short paragraph, the near facts:[1]

> Analytically the hyperbola is given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ wherein $ab > h^2$. $\cdots$ In the rectangular hyperbola $a = b$; hence its equation is $x^2 - y^2 = 0$ $\cdots$.

How can we believe and teach the doctrine that (as Bacon might have said it) "Mathematicks maketh the careful man" if such inaccurate statements are permitted in books which the public has a right to regard as sources of exact information?

III. Geometric Proofs of the Law of Tangents.

By H. C. Bradley, Massachusetts Institute of Technology.

Let ABC be the given triangle, $\angle A$ acute and greater than $\angle B$. Produce AC. Lay off $\angle ABD = \angle A$ on the same side of AB as C, forming the isosceles triangle ABD. Let E be the middle point of AB. Draw DE.

Produce BC. Lay off $CF = AC$. Draw AF, thus forming the isosceles triangle ACF. Let G be the middle point of AF. Draw GC, and produce it to meet DE at O.

Connect O with B. From O draw OH perpendicular to BC. Then

$$OH = BH \tan \angle OBH = CH \tan \angle OCH. \qquad (1)$$

In the triangle BCD, CO and DO bisect the angles at C and D respectively. Hence BO bisects the angle at B. Whence $\angle OBH = \frac{1}{2}(A - B)$. Also, we have $\angle OCH = \frac{1}{2}(A + B)$.

In the triangle ABF, OE and OG are, respectively, perpendicular bisectors of the sides AB and AF. Hence OH is the perpendicular bisector of BF, and $BH = HF$. Whence $BH = \frac{1}{2}(a + b)$, and $CH = \frac{1}{2}(a - b)$.

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[1] Vol. 14, p. 199.